22/20/25. IPE-307

L-3/T-1/IPE Date: 24/08/2025

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-1 B. Sc. Engineering Examinations 2023-2024

Sub: **IPE 307** (Operations Research)

Full Marks: 280 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

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## SECTION – A

There are **FOUR** questions in this section. Question No. 1 is **COMPULSORY**. Answer any **TWO** from the rest.

1. (a) As for Property 3 of the exponential distribution, let $T_1, T_2, ..., T_n$ be independent exponential random variables with parameters $a_1, a_2, ..., a_n$, respectively and let $U = \min\{T_1, T_{2...,} T_n\}$. Show that the probability that a particular random variable $T_j$ will turn out to be smallest of the $n$ random variables is. **(8)**

$$p\{T_j = U\} = a_j \Big/ \sum_{i=1}^{n} a_i, \quad \text{for } j = 1, 2, ..., n.$$

(b) Suppose that all car owners fill up when their tanks are exactly half full. At the present time, an average of 7.5 customers per hour arrive at a single-pump gas station. It takes an average of 4 minutes to service a car. Assume that interarrival times and service times are both exponential. **($20\frac{2}{3}$) (CO3)**

i) For the present situation, compute $L$ and $W$.

ii) Suppose that a gas shortage occurs and panic buying takes place. To model this phenomenon, suppose that all car owners now purchase gas when their tanks are exactly three-quarters full. Since each car owner is now putting less gas into the tank during each visit to the station, we assume that the average service time has been reduced to $3^{1/3}$ minutes. How has panic buying affected $L$ and $W$?

(c) Derive the expressions for the expected number of customers in queueing system for the finite queue variation of the $M/M/s$ model. **(18)**

2. (a) Write the KKT necessary conditions and solve them for the problem **(26)**

$$\text{Minimize } F(x_1, x_2) = 2x_1 + x_2 - x_1^2 - x_2^2 - 2$$

$$\text{Subject to } 2x_1 + x_2 \geq 4, x_1 + 2x_2 \geq 4$$

Contd .......... P/2

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**(Contd ... Q. No. 2)**

(b) Consider the nonlinear problem: **(20⅔)**

$$\min \quad f(x) = x_1^2 + x_2^2 - 4x_1 + 4$$
$$s.t.$$
$$g_1(x) = x_1 - x_2 + 2 \geq 0$$
$$g_2(x) = -x_1^2 + x_2 - 1 \geq 0$$
$$g_3(x) = x_1 \geq 0$$
$$g_4(x) = x_2 \geq 0$$

Show that the constraints define a convex set. Also, show that the objective function *f(x)* is convex.

3. (a) In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale. **(10)**

i) Find the probability that the grandson of a man from Harvard went to Harvard.

ii) Modify the above by assuming that the son of a Harvard man always went to Harvard. Again, find the probability that the grandson of a man from Harvard went to Harvard.

(b) Write the linear programming formulations of the two-person zero-sum game for both players and show that they are dual of each other and thereby prove the minimax theorem. **(10)**

(c) The Research and Development Division of the Progressive Company has been developing four possible new product lines. Management must now make a decision as to which of these four products actually will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix.
A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the product mix that maximizes the total profit (total net revenue minus start-up costs). **(16⅔)**

| | Product | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Start-up cost ($) | 50,000 | 40,000 | 70,000 | 60,000 |
| Marginal revenue ($) | 70 | 60 | 90 | 80 |

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**(Contd ... Q. No. 3(c))**

Let the continuous decision variables $x_1, x_2, x_3$ and $x_4$ be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

i) No more than two of the products can be produced.

ii) Either product 3 or 4 can be produced only if either product 1 or 2 is produced.

*iii) Either* $5x_1 + 3x_2 + 6x_3 + 4x_4 \le 6{,}000$

*or* $4x_1 + 6x_2 + 3x_3 + 5x_4 \le 6{,}000$

Introduce auxiliary binary variables to formulate a mixed BIP model for this problem. (Do not solve)

(d) Briefly discuss two application of dynamic programming with suitable examples. **(10)**

4. (a) Distinguish between *discrete-event* and *continuous simulations* using suitable examples. **(8)**

(b) What is continuous time Markov chain? Explain why the birth-and-death process is special type of continuous time Markov chin. **(8)**

(c) Consider the general *mxn*, two-person, zero-sum game. Let $p_{ij}$ denote the payoff to player 1 if he plays his strategy $i(i = 1,\ldots,m)$ and player 2 plays her strategy $j(j = 1,\ldots,n)$. Strategy 1 (say) for player 1 is said to be *weakly dominated* by strategy 2 (say) if $p_{1j} \le p_{2j}$ for $j = 1,\ldots,n$ and $p_{1j} = p_{2j}$ for one or more values of $j$. **($30\frac{2}{3}$)**

i) Assume that the payoff table possesses one or more saddle points, so that the players have corresponding optimal pure strategies under the minimax criterion. Prove that eliminating weakly dominated strategies ~~form~~ from the payoff table cannot eliminate all these saddle points and cannot produce any new ones.

ii) Assume that the payoff table does not possess any saddle points, so that the optimal strategies under the minimax criterion are mixed strategies. Prove that eliminating weakly dominated pure strategies from the payoff table cannot eliminate all optimal mixed strategies and cannot produce any new ones.

Contd ......... P/4

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## SECTION – B

There are **FOUR** questions in this section. **Question No. 5 is COMPULSORY.** Answer any **TWO** from the rest. Assume any reasonable value for missing data.

5. (a) GatroDelights manufactures artisanal cheese and gourmet chocolate using milk, cocoa, and cream. Currently, 40 lb of milk, 30 lb of cocoa, and 40 lb of cream are available. One unit of artisanal cheese sells for \$40 and requires 1 lb of milk, 1 lb of cocoa, and 2 lb of cream. One unit of gourmet chocolate sells for \$50 and requires 2 lb of milk, 1 lb of cocoa, and 1 lb of cream. GastroDelights can sell all artisanal cheese and gourmet chocolate that are produced. To maximize total sales revenue, GastroDelights should solve the following LP: **(40)** **(CO1)**

Max $z = 40\text{ Cheese} + 50\text{ Chocolate}$

Subject to:

Milk : Cheese + 2 Chocolate $\leq 40$

Cocoa: Cheese + Chocolate $\leq 30$

Cream: 2 Cheese + Chocolate $\leq 40$

Cheese $\geq 0$, Chocolate $\geq 0$

Here,

Cheese = number of artisanal cheese units produced
Chocolate = number of gourmet chocolate units produced

Following table is the optimal table for the problem:

| z | Cheese | Chocolate | $S_1$ | $S_2$ | $S_3$ | RHS |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 20 | 0 | p | q |
| 0 | 0 | 1 | $\frac{2}{3}$ | 0 | $-\frac{1}{3}$ | $\frac{40}{3}$ |
| 0 | 0 | 0 | $-\frac{1}{3}$ | 1 | $-\frac{1}{3}$ | $\frac{10}{3}$ |
| 0 | 1 | 0 | $-\frac{1}{3}$ | 0 | $\frac{2}{3}$ | $\frac{40}{3}$ |

Here, S1, S2, S3 are slacks associated with constraits.

Use this optimal tableau to answer the following questions.

i) Find the optimal solution for Cheese and Chocolate from the table. Also calculate the optimum revenue.

Contd .......... P/5

ii) Write down the dual to GatroDelight' LP and find its optimal solution.

iii) Identify the interval of permissible values for the unit price of artisanal cheese over which the current basis remains optimal.

iv) Identify the interval of permissible values for the unit price of gourmet chocolate over which the current basis remains optimal.

v) Find the range of values of the amount of available milk for which the current basis remains optimal.

vi) Find the range of values of the amount of available cocoa for which the current basis remains optimal.

vii) Find the range of values of the amount of available cream for which the current basis remains optimal.

viii) Suppose GatroDelights is considering manufacturing a new product: cream cheese. One unit of cream cheese requires 0.5 lb of milk, 3 lb of cocoa and 3 lb of cream and sells for $50. Should GatroDelights manufacture any cream cheese?

(b) Briefly discuss the big advantage of reoptimization technique over re-solving from scratch during postoptimality analysis. $(6\frac{2}{3})$

(CO2)

6. (a) Consider the following LP model: (18)

$$\text{Maximize } z = 5x_1 + 2x_2 + 3x_3$$

Subject to:

$$x_1 + 5x_2 + 2x_3 \le b_1$$
$$x_1 - 5x_2 - 6x_3 \le b_2$$
$$x_1, x_2, x_3 \ge 0$$

The following optimal tableau corresponds to specific values of $b_1$ and $b_2$:

| Basic | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | Solution |
|---|---|---|---|---|---|---|
| $z$ | 0 | a | 7 | d | e | 150 |
| $x_1$ | 1 | b | 2 | 1 | 0 | 30 |
| $x_5$ | 0 | c | -8 | -1 | 1 | 10 |

Determine the following:

i) The right-hand-side values, $b_1$ and $b_2$.

ii) The optimal dual solution.

iii) The elements a, b, c

iv) Construct the dual problem using SOB method.

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**(Contd ... Q. No. 6)**

(b) Consider the three-dimensional LP solution space in Figure-6b, whose feasible extreme points are A, B, .........., and J. **(18)**

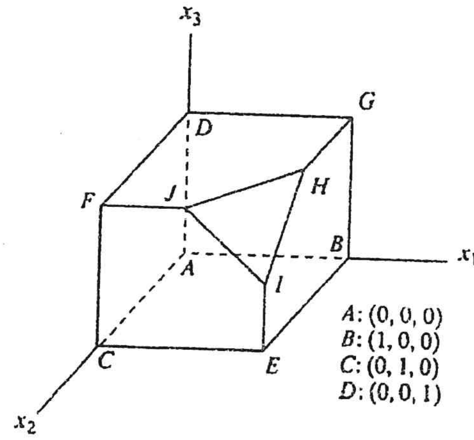


**Figure-6b**: Solution Space

Required:

i) Suppose, that the simplex iterations start at A and the optimum occurs at H. Indicate whether any of the following paths are not legitimate for the simplex algorithm, and state the reason.

- A → B → G → H
- A → C → I → H
- A → C → E → B → A → D → G → H

ii) For the solution space in figure-6b, all the constraints are of the type ≤ and all the variables $x_1$, $x_2$ and $x_3$ are nonnegative. Suppose, that $s_1, s_2, s_3$ and $s_4$ $(\geq 0)$ are the slacks associated with constraints represented by the planes CEIJF, BEIHG, DFJHG and IJH respectively. Identify the basic variables associated with each feasible extreme point of the solution space.

(c) Define shadow price. Also, discuss its relationship with the complementary optimal solutions property. **($10\frac{2}{3}$)**

7. (a) Show & Sell can advertise its products on local radio and television (TV), or in newspapers. The advertising budget is limited to \$10,000 a month. Each minute of advertising on radio costs \$15 and each minute on TV costs \$300. A newspaper ad costs \$50. Show & sell likes to advertise on radio at least twice as much as on TV. In the meantime, the use of at least 5 newspaper ads and no more than 400 minutes of radio advertising a month is recommended. Past experience shows that advertising on TV is 50 times more effective than on radio and 10 times more effective than in newspapers. Show & Sell wants to maximize total adverting effectiveness. **(21)**

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**(Contd ... Q. No. 7 (a))**

Required:

i) Formulate the problem as a linear program.

ii) Determine the optimum allocation of the budget to the three media using Simplex Method.

iii) If the monthly budget is increased by 50%, would this result in proportionate increase in the overall effectiveness of advertising?

(b) Consider the following problem.

Maximize $Z = -2x_1 + x_2 - 4x_3 + 3x_4$

Subject to

$$x_1 + x_2 + 3x_3 + 2x_4 \leq 4$$
$$x_1 - x_3 + x_4 \geq -1$$
$$2x_1 + x_2 \leq 2$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 2$$

and

$$x_1, x_2, x_3, x_4 \geq 0$$

Using the two-phase method, form the initial simplex tableau for Phase II and restore the proper form.

(c) "From the practical standpoint, the condition reveals that the model has at least one redundant constraint" - justify the statement for a special case that arises in the simplex method. (10⅔)

8. (a) Consider the transportation problem having the following parameter table: (25)

| | | Destination | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | Supply |
| Source | 1 | 2 | 4 | 6 | 5 | 7 | 4 |
| | 2 | 7 | 6 | 3 | M | 4 | 6 |
| | 3 | 8 | 7 | 5 | 2 | 5 | 6 |
| | 4 | 0 | 0 | 0 | 0 | 0 | 4 |
| Demand | | 4 | 4 | 2 | 5 | 5 | |

Use each of the following criteria to obtain an initial BF solution. Compare the values of the objective function for these solutions.

i) Northwest corner rule

ii) Vogel's approximation method.

iii) Russell's approximation method.

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**(Contd ... Q. No. 8)**

(b) Five workers are available to perform four jobs. The time it takes each worker to perform each job is given in following table. The goal is to assign workers to jobs so as to minimize the total time required to perform the four jobs. Use the Hungarian method to solve the problem. **(15)**

| Worker | Time (Hours) | | | |
|---|---|---|---|---|
| | Job 1 | Job 2 | Job 3 | Job 4 |
| 1 | 10 | 15 | 10 | 15 |
| 2 | 12 | 8 | 20 | 16 |
| 3 | 12 | 9 | 12 | 18 |
| 4 | 6 | 12 | 15 | 18 |
| 5 | 16 | 12 | 8 | 12 |

(c) Classify the possible types of basic solutions that may result from sensitivity analysis in linear programming, and specify the appropriate reoptimization method to apply in each case. $(6\frac{2}{3})$

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